November 15th, 2013 **Name (Please Print)** _____

Differential Equations - Final - Semester I 13/14

Your Signature _____

1. Find the general solution to the differential equation

$$\frac{dx}{dt}(t) = ax(t) + be^{-\lambda t}, t > 0$$

Let $a < 0$, $\lambda > 0$ and $b \in \mathbb{R}$. Decide whether a solution exists and if it does then find the limit as $t \to \infty$?

2. Consider the second order linear differential equation:

$$3\frac{d^2y}{dt^2} - 15\frac{dy}{dt} + 18y = 0$$

(a) Find the general solution.

(b) Find the particular solution satisfying $y(0) = 0, y'(0) = \alpha$.

(c) When is $\lim_{t\to\infty} y(t) = 0$?

3. Solve the following short answer questions:

(a) Let $f : [0, \infty) \times \mathbb{R} \to \mathbb{R}$ be a continuous function. Prove or Disprove: The initial value problem

$$\frac{dy}{dt} = f(t, y) \text{ for } t > 0, y(0) = 0.$$

has a unique solution $y : [0, \infty) \to \mathbb{R}$.

(b) Consider the second order linear differential equation:

$$\frac{d^2y}{dt^2} + p(t)\frac{dy}{dt} + q(t)y + r(t) = 0,$$

i. Suppose $r \equiv 0$ and p, q having one regular singularity at 0. Write out the indical equation and possible Frobenius series solutions for the homogeneous ODE.

ii. Suppose ϕ_1 and ϕ_2 are two linearly independent solutions of the corresponding homogeneous ODE. Then write down the general solution for the above.

4. Using separation of variables technique solve the following boundary value problem:

$$\begin{aligned} &u_t = 9u_{xx} - 10u \quad \text{if } 0 \le x \le 1, t \ge 0 \\ &u(0,t) = 0, u(1,t) = 0, \text{ if } t \ge 0, \\ &u(x,0) = \sin(4\pi x) \text{ if } 0 \le x \le 1. \end{aligned}$$

5. Solve the Dirichlet Problem given by

$$\begin{aligned} \Delta u(x) &= 0 \text{ if } 0 \le \mid x \mid < 1 \\ u(x) &= 100 \text{ if } \mid x \mid = 1, \ x_2 \ge 0 \\ u(x) &= 0 \text{ if } \mid x \mid = 1, \ x_2 < 0 \end{aligned}$$

(you may assume that the problem has a unique solution).